



Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Federal Institute of Metrology METAS



OIML R87 Quantity of prepackages: Statistics

Hans-Peter Vaterlaus, formerly METAS, Switzerland

OIML R 87: Edition 2016

This Recommendation specifies requirements for the **quantity of product in prepackages**:

- which are labeled in predetermined constant nominal quantities of weight, volume, length, area and count;
- Specifies sampling plans and procedures for use by legal metrology officials in verifying the quantity of product in prepackages

Note: R 87 covers only prepackages of equal nominal content !

OIML R 87: Metrological requirements (1)

Nominal quantity Q_{nom} in g or ml	Value of T in % of Q_{nom}	Value of T in g or ml
0 to 50	9	-
50 to 100	-	4.5
100 to 200	4.5	-
200 to 300	-	9
300 to 500	3	-
500 to 1'000	-	15
1'000 to 10'000	1.5	-
10'000 to 15'000	-	150
Above 15'000	1	-

- (1) **Average requirements:** On average, the quantity in prepackages shall at least be equal to the nominal quantity Q_{nom}
- (2) **Individual requirements:** Only a small percentage of the prepackages (typ. 2.5 %) are allowed to have a quantity between $(Q_{nom} - 2T)$ and $(Q_{nom} - T)$, called *T1* error.
- (3) No prepackage shall have a quantity less than $Q_{nom} - 2T$ (referred to as *T2* error).

OIML R 87: Metrological requirements (2)

Nominal quantity of product in length	Percent of Q_{Nom}
$Q_{nom} \leq 5\ m$	No tolerable deficiency allowed
$Q_{nom} > 5\ m$	2

Nominal quantity of product in area	Percent of Q_{Nom}
All Q_{nom}	3

Nominal quantity of product in count	Percent of Q_{Nom}
$Q_{nom} \leq 50\ items$	No tolerable deficiency allowed
$Q_{nom} > 50\ items$	1 (rounded up)

OIML R 87: Metrological requirements (3)

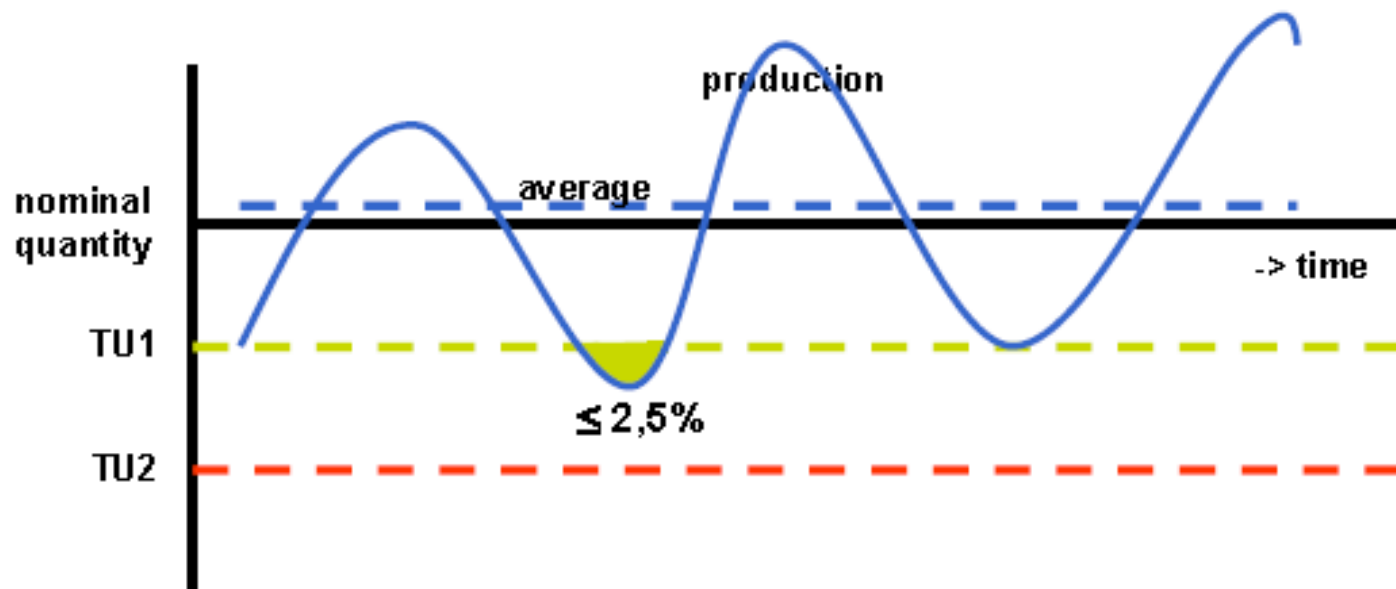
The so called «3-packers rules»:

- In average, the quantity of product in prepackages shall not be less than the declared nominal quantity Q_{nom}
- Only a small amount of prepackages (typ. 2.5 %) are allowed to show deficiencies between $T1$ and $T2$.
- No prepackage shall have a quantity less than $Q_{nom} - 2T$ (referred to as $T2$ error)



OIML R 87: Metrological requirements (4)

- The average quantity in a batch (1 h production) shall not be less than Q_{nom} .
- TU1 (or T1) : only few prepackages (typ. $\leq 2.5\%$) shall exceed a T1 error
- TU2 (or T2): no prepackages allowed to exceed the TU2 (or T2) error.



OIML R 87: Statistical approach

The tests are carried out by statistical means, based on random sampling of an hourly lot of production with lot size N and sample size n .

Average requirements for sampled lots:

- The probability of incorrectly rejecting an inspection lot satisfying equation $\mu \geq Q_{nom}$ shall be no more than 0.5 % (Producer's risk, PR).
- The probability of correctly rejection an inspection lot shall be at least 90 % (Consumer's risk, CR)

Individual requirements for sampled lots:

- The probability of incorrectly rejecting a lot with 2.5 % of the prepackages having $T1$ or $T2$ errors shall be smaller than 5 % (PR).
- The probability of correctly rejecting an inspection lot with 9 % of the prepackages having $T1$ or $T2$ errors shall be at least 90 % (CR).

OIML R 87: Test of average requirements (1)

Reject the lot if $e_{ave}/s + SCF < 0$ with $e_{ave} = \frac{1}{n} \sum_{i=1}^n e_i$

e_{ave} is the average of errors and e_i are the individual prepackage errors defined by $e_i = q_i - Q_{nom}$, taking into account their signs, n is the number of prepackages of the sample size, s is the sample standard deviation of the individual errors, and SCF is the Sample Correction Factor.

Please note the difference between e_{ave} and E_{ave}

e_{ave} : means average error for prepackages of sample size n

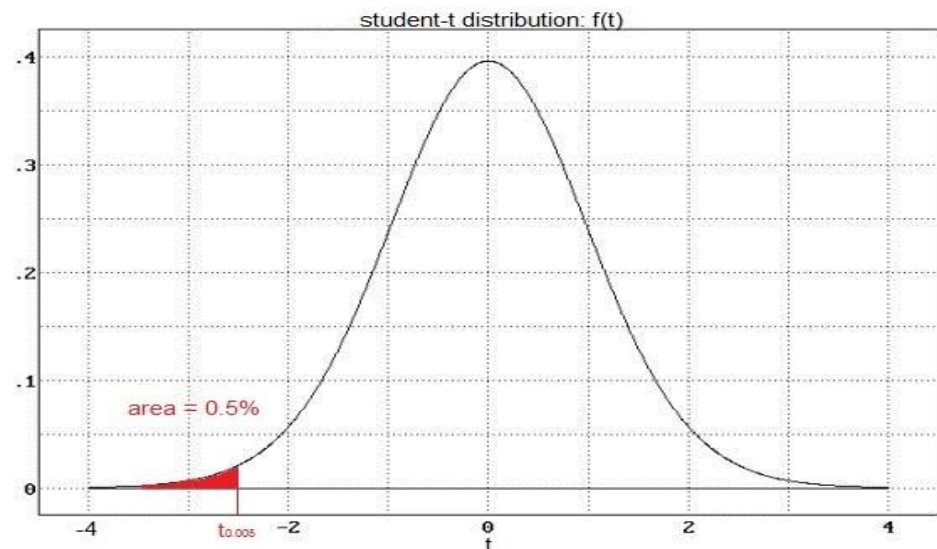
E_{ave} : means average error for prepackages of lot size N

OIML R 87: Test of average requirements (2)

The SCF for the defined lot size N and sample size n is calculated using the formula:

$$SCF = - \frac{t_{0.005, n-1}}{\sqrt{\frac{n(N-1)}{N-n}}}$$

$t_{0.005, n-1}$ is the quantile of the Student's t inverse cumulative function with probability $p = 0.005$ and degree of freedom $f = n-1$



$(N-n)/(N-1)$ is the so called finite population correction factor

OIML R 87: Test of average requirements (3)

Please note:

$$e_{ave}/s + SCF < 0 \quad \text{with} \quad e_{ave} = \frac{1}{n} \sum_{i=1}^n e_i$$

can also be formulated or rewritten in the following way
(which is often easier to be understand):

Reject the lot if: $q_{ave} < Q_{Nom} - SCF \cdot s$ with

q_{ave} : mean value of actual quantities q_i in a sample n

Q_{Nom} : Nominal quantity declared on the label

OIML R 87: calculations of SCF (1)

Example of calculation of the sample correction factor SCF by using a lot size of $N = 100$ and sample size $n = 49$

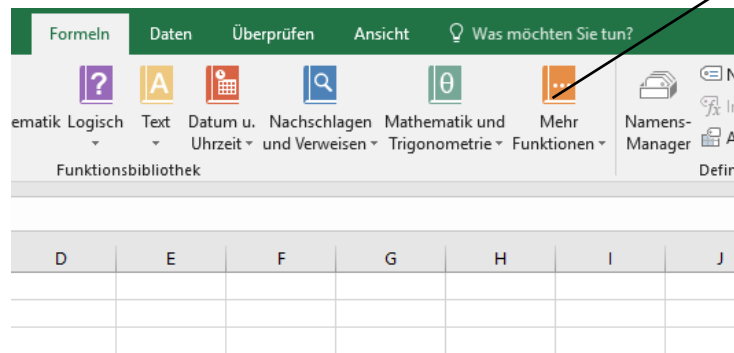
Procedure:

1. Use EXCEL and calculate $t_{p,n-1}$ with $p = 0.005$ and $n = 49$

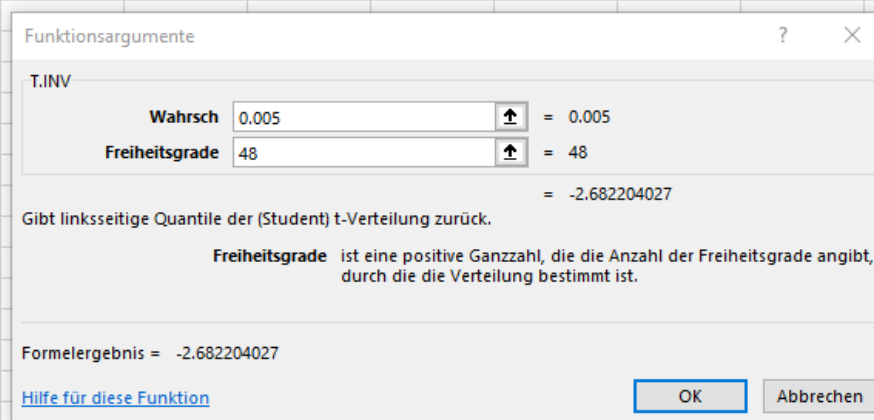
2. Calculate SCF according:
$$SCF = - \frac{t_{0.005,n-1}}{\sqrt{\frac{n(N-1)}{N-n}}}.$$

OIML R 87: calculations of SCF (2)

Calculate $t_{p,n-1}$ with $p = 0.005$ and $n = 49$



choose Statistics
then choose T.INV



Excel calculates the
value of $t_{p,n-1} = -2.6822$
with $p = 0.5\%$ and $n-1 = 48$

OIML R 87: calculations of SCF (3)

Once the value of $t_{p,n-1}$ has been calculated for the probability $p = 0.5 \%$ and a given sample size n , the sampling correction factor SCF can be calculated using the formula:

$$SCF = - \frac{t_{0.005,n-1}}{\sqrt{\frac{n(N-1)}{N-n}}}.$$

for a given lot size N and sample size n .

In our case, with $N = 100$ and $n = 49$ the value of SCF will be:

$$\mathbf{SCF = 0.275}$$

OIML R 87: calculations of SCF (4)

- **Example 1**

Calculate SCF for $N = 500$ and $n = 81$

- **Example 2**

Calculate SCF for $N = 20'000$ and $n = 98$

- **Example 3**

Calculate SCF for $N = 50'000$ and $n = 98$

OIML R 87: Test of individual requirements

The test for the two individual requirements is realized by performing statistics for the values of n_{T1} and n_{T2} .

- The number n_{T1} is the maximum number of samples in the sample size n being allowed for accepting the lot under the assumption that only 2.5 % of prepackages of the lot with size N are allowed to have a T1 error.
- The lot has to be rejected for n_{T2} being larger than zero, this means no prepackage shall have a quantity less than $Q_{nom} - 2T$.

OIML R 87: Single step sampling plan

Sampling plan for lot sizes N based on the statistical requirements (average and individual requirements):

Inspection lot size N	Sample size n	SCF	Number of allowed $T1$ errors n_{T1}
$N \leq 20$	100 % inspection	NA	0
40	32	0.22	1
60	35	0.30	1
80	47	0.25	2
100	49	0.28	2
200	64	0.27	3
300	67	0.29	3
400	81	0.26	4
500	81	0.27	4
600 – 30'000	98	0.24 – 0.26	5
> 30'000	98	0.27	5

OIML R 87: Multistep sampling plan

The stepwise sampling plan is a new feature in the OIML R 87 Edition 2016

Why a stepwise sampling plan?

- Decreases the burden on packers by requiring fewer samples to be taken for control inspections.
- Saves time and costs for inspectors as well as packers since fewer prepackages have to be opened and destroyed for control purposes
- Particularly beneficial for inspectors in checking high volume production of prepacked goods

OIML R 87: Multistep sampling plan for lot size $N = 120$ and 4 steps

Step number	Cumulative sample size n	SCF	Number of allowed $T1$ errors n_{T1}
1	35	0.39	0
2	50	0.29	1
3	60	0.24	2
4	75	0.19	3

OIML R 87: Multistep sampling plan for lot size $N = 120$ and 4 steps

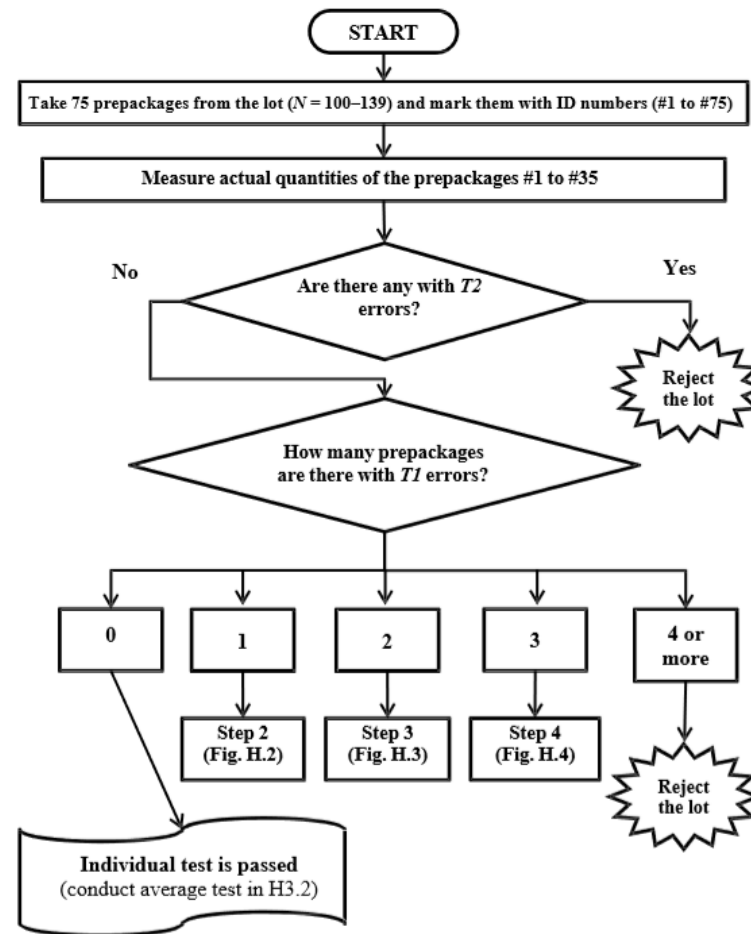


Figure H.1 Stepwise sampling method to test the individual requirement for a lot size $N = 100-139$
Step 1: Start the entire procedure

OIML R 87: Multistep sampling plan for lot size $N = 50'000$ and 7 steps

Step number	Cumulative sample size n	SCF	Number of allowed $T1$ errors n_{T1}
1	40	0.43	0
2	55	0.36	1
3	70	0.32	2
4	95	0.27	3
5	105	0.26	4
6	120	0.24	5
7	135	0.22	6

OIML R 87: Multistep sampling plan

Typical Application:

- For producers with high hourly production rate of prepackages (eg Chocolate manufacturer)
- Manufacturer with low risk of underfilling prepackages
- Checks of prepackages at stock or warehouses by legal metrology officials





Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Federal Institute of Metrology METAS



Thank you very much for your attention