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OIML R87 Quantity of prepackages: Statistics

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OIML R 87: Edition 2016

This Recommendation specifies requirements for the **quantity of product in prepackages**:

➢ which are labeled in predetermined <u>constant nominal quantities</u> of weight, volume, length, area and count;

➢ Specifies sampling plans and procedures for use by legal metrology officials in verifying the quantity of product in prepackages

Note: R 87 covers only prepackages of equal nominal content !

OIML R 87: Metrological requirements (1)

Nominal quantity Q_{nom} in g or ml	Value of T in % of Q_{nom}	Value of T in g or ml
0 to 50	9	-
50 to 100	-	4.5
100 to 200	4.5	-
200 to 300	-	9
300 to 500	3	-
500 to 1'000	-	15
1'000 to 10'000	1.5	-
10'000 to 15'000	-	150
Above 15'000	1	-

(1) Average requirements: On average, the quantity in prepackages shall at least be equal to the nominal quantity Q_{nom}

(2) Individual requirements: Only a small percentage of the prepackages (typ. 2.5 %) are allowed to have a quantity between $(Q_{nom} - 2T)$ and $(Q_{nom} - T)$, called *T1* error.

(3) No prepackage shall have a quantity less than $Q_{nom} - 2T$ (referred to as *T2* error).

Nominal quantity of product in length	Percent of Q_{Nom}
$Q_{nom} \leq 5\ m$	No tolerable defiency allowed
$Q_{nom} > 5\ m$	2

Nominal quantity of product in area	Percent of Q_{Nom}
All Q_{nom}	3

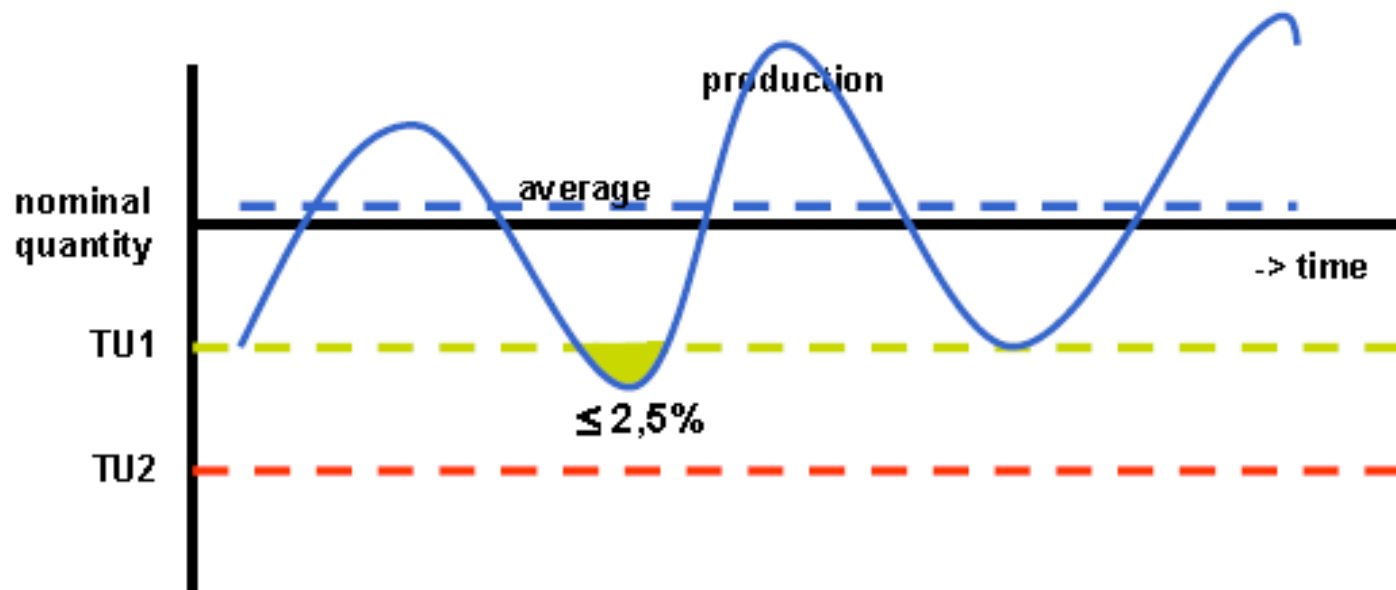| Nominal quantity of product in count	Percent of Q_{Nom}
$Q_{nom} \leq 50\ items$	No tolerable defiency allowed
$Q_{nom} > 50\ items$	1 (rounded up)

OIML R 87: Metrological requirements (3)

The so called «3-packers rules»:

➤ In average, the quantity of product in prepackages shall not be less than the declared nominal quantity Q_{nom}

➤ Only a small amount of prepackages (typ. 2.5 %) are allowed to show deficiences between T1 and T2.

➤ No prepackage shall have a quantity less than $Q_{nom} - 2T$ (referred to as *T2* error)

- The average quantity in a batch (1 h production) shall not be less than Q_{nom}.
- TU1 (or T1) : only few prepackages (typ. ≤ 2.5 %) shall exceed a T1 error
- TU2 (or T2): no prepackages allowed to exceed the TU2 (or T2) error.

Figure 1 – Example of *T1* and *T2* Errors for *T* = 4.5 g

	100 g	Nominal quantity (Q_{nom})	
Individual package errors (E_i) (inadequate prepackage)	$-T \leq E_i < 0$ $-4.5\ g \leq E_i < 0$	Individual package quantities Q_i less than Q_{nom} but equal to or greater than $(Q_{nom} - T)$ are acceptable variations.	
	$-2T \leq E_i < -T$ $-9\ g \leq E_i < -4.5\ g$	↑ *T1* Error ↓	Individual package errors less than $-T$ but equal to or greater than $-2T$ are called *T1* errors.
	$E_i < -2T$ $E_i < -9\ g$	↑ *T2* Error ↓	Individual package errors less than $-2T$ are called *T2* errors.

OIML R 87: Statistical approach

The tests are carried out by statistical means, based on random sampling of an hourly lot of production with lot size N and sample size n.

Average requirements for sampled lots:

- The probability of incorrectly rejecting an inspection lot satisfying equation $\mu \geq Q_{nom}$ shall be no more than 0.5 % (Producer's risk, PR).
- The probability of correctly rejection an inspection lot shall be at least 90 % (Consumer's risk, CR)

Individual requirements for sampled lots:

- The probability of incorrectly rejecting a lot with 2.5 % of the prepackages having *T1* or *T2* errors shall be smaller than 5 % (PR).
- The probability of correctly rejecting an inspection lot with 9 % of the prepackages having *T1* or *T2* errors shall be at least 90 % (CR).

Reject the lot if $e_{ave}/s + SCF < 0$ with $e_{ave} = \dfrac{1}{n}\sum_{i=1}^{n} e_i$

e_{ave} is the average of errors and e_i are the individual prepackage errors defined by $e_i = q_i - Q_{nom}$, taking into account their signs, n is the number of prepackages of the sample size, s is the sample standard deviation of the individual errors, and SCF is the Sample Correction Factor.

Please note the difference between e_{ave} and E_{ave}

e_{ave}: means average error for prepackages of <u>sample size n</u>

E_{ave}: means average error for prepackages of <u>lot size N</u>

The SCF for the defined lot size N and sample size n is calculated using the formula:

$$SCF = -\frac{t_{0.005,n-1}}{\sqrt{\dfrac{n(N-1)}{N-n}}}.$$

$t_{0.005,n-1}$ is the quantile of the Student's t inverse cumulative function with probability p = 0.005 and degree of freedom f = n-1

student-t distribution: f(t)

area = 0.5%
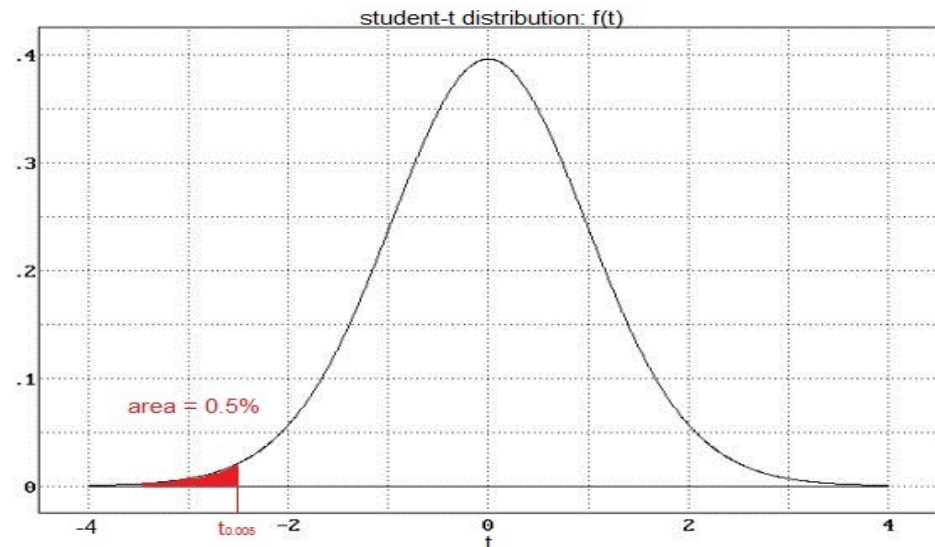
(N-n)/(N-1) is the so called finite population correction factor

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Please note:

$$e_{ave}/s + SCF < 0 \quad \text{with} \quad e_{ave} = \frac{1}{n}\sum_{i=1}^{n} e_i$$

can also be formulated or rewritten in the following way (which is often easier to be understand):

Reject the lot if: $q_{ave} < Q_{Nom} - SCF \cdot s$ with

q_{ave} : mean value of actual quantities q_i in a sample n

Q_{Nom} : Nominal quantity declared on the label

Example of calculation of the sample correction factor SCF by using a lot size of N = 100 and sample size n = 49
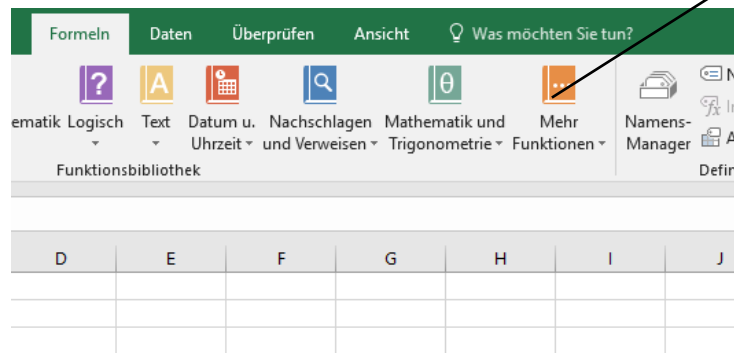
Procedure:

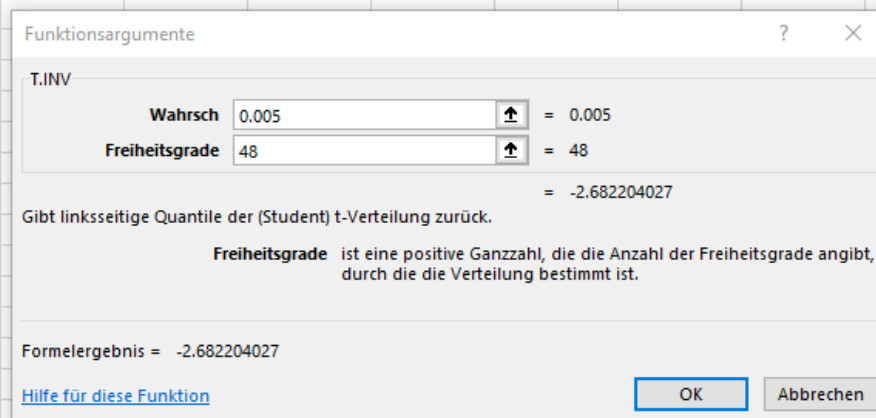1. Use EXCEL and calculate $t_{p,n-1}$ with p = 0.005 and n= 49

2. Calculate SCF according:
$$SCF = -\frac{t_{0.005,n-1}}{\sqrt{\dfrac{n(N-1)}{N-n}}}.$$

OIML R 87: calculations of SCF (2)

Calculate $t_{p,n-1}$ with p = 0.005 and n= 49

choose Statistics
then choose T.INV

Excel calculates the
value of $t_{p,n-1}$ = - 2.6822
with p = 0.5% and n-1 = 48

Once the value of $t_{p,n-1}$ has been calculated for the probability p = 0.5 % and a given sample size n, the sampling correction factor SCF can be calculated using the formula:

$$SCF = -\frac{t_{0.005,n-1}}{\sqrt{\dfrac{n(N-1)}{N-n}}}.$$

for a given lot size N and sample size n.

In our case, with N = 100 and n = 49 the value of SCF will be:

SCF = 0.275

OIML R 87: calculations of SCF (4)

- **Example 1**
  Calculate SCF for N = 500 and n = 81

- **Example 2**
  Calculate SCF for N = 20'000 and n = 98

- **Example 3**
  Calculate SCF for N = 50'000 and n = 98

OIML R 87: Test of individual requirements

The test for the two individual requirements is realized by performing statistics for the values of n_{T1} and n_{T2}.

- The number n_{T1} is the maximum number of samples in the sample size n being allowed for accepting the lot under the assumption that only 2.5 % of prepackages of the lot with size N are allowed to have a T1 error.

- The lot has to be rejected for n_{T2} being larger than zero, this means no prepackage shall have a quantity less than $Q_{nom} - 2T$.

OIML R 87: Single step sampling plan

Sampling plan for lot sizes N based on the statistical requirements (average and individual requirements):

Inspection lot size N	Sample size n	SCF	Number of allowed $T1$ errors n_{T1}
N ≤ 20	100 % inspection	NA	0
40	32	0.22	1
60	35	0.30	1
80	47	0.25	2
100	49	0.28	2
200	64	0.27	3
300	67	0.29	3
400	81	0.26	4
500	81	0.27	4
600 – 30'000	98	0.24 – 0.26	5
> 30'000	98	0.27	5

OIML R 87: Multistep sampling plan

The stepwise sampling plan is a new feature in the OIML R 87 Edition 2016

Why a stepwiese sampling plan?

➢ Decreases the burden on packers by requiring fewer samples to be taken for control inspections.

➢ Saves time and costs for inspectors as well as packers since fewer prepackages have to be opened and destroyed for control purposes

➢ Particularly beneficial for inspectors in checking high volume production of prepacked goods

OIML R 87: Multistep sampling plan for lot size N = 120 and 4 steps   (Annex H)

Step number	Cumulative sample size n	SCF	Number of allowed T1 errors n_{T1}
1	35	0.39	0
2	50	0.29	1
3	60	0.24	2
4	75	0.19	3

OIML R 87: Multistep sampling plan for lot size N = 120 and 4 steps   (Annex H)
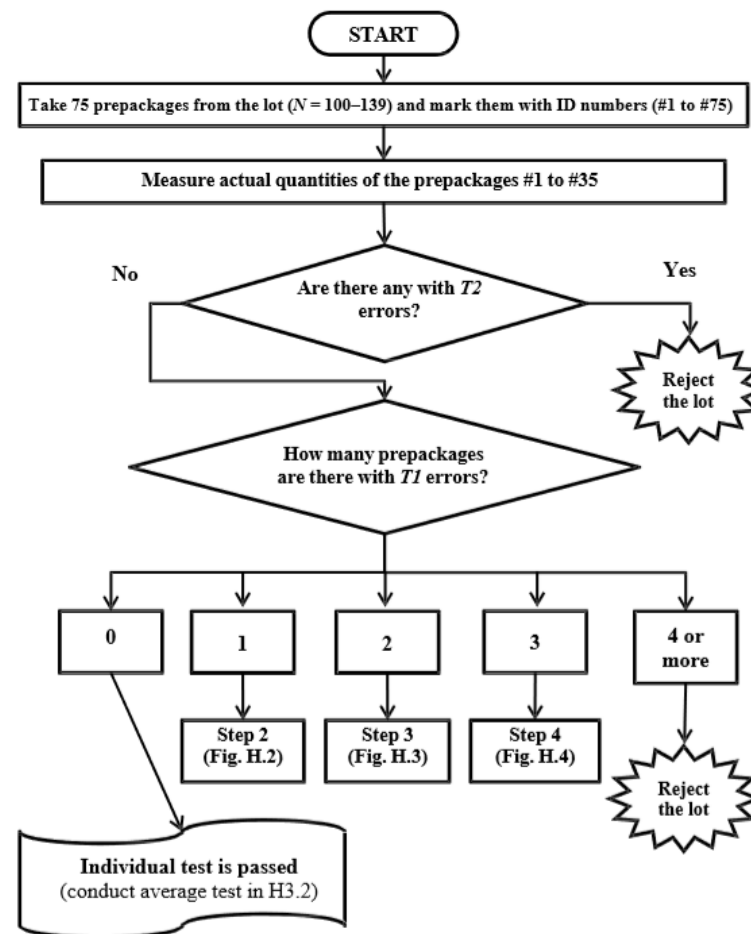

Figure H.1   Stepwise sampling method to test the individual requirement for a lot size
$N = 100–139$
Step 1: Start the entire procedure

OIML R 87: Multistep sampling plan for lot size N = 50'000 and 7 steps (Annex H)

Step number	Cumulative sample size n	SCF	Number of allowed $T1$ errors n_{T1}
1	40	0.43	0
2	55	0.36	1
3	70	0.32	2
4	95	0.27	3
5	105	0.26	4
6	120	0.24	5
7	135	0.22	6

OIML R 87: Multistep sampling plan

Typical Application:

➢ For producers with high hourly production rate of prepackages (eg Chocolate manufacturer)

➢ Manufacturer with low risk of underfilling prepackages

➢ Checks of prepackages at stock or warehouses by legal metrology officials

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Thank you very much for your attention