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OIML R87 Quantity of prepackages: Statistics

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OIML R 87: Edition 2016

This Recommendation specifies requirements for the **quantity of product in prepackages:**

- which are labeled in predetermined constant nominal quantities of weight, volume, length, area and count;
- Specifies sampling plans and procedures for use by legal metrology officials in verifying the quantity of product in prepackages

Note: R 87 covers only prepackages of equal nominal content !

OIML R 87: Metrological requirements (1)

Nominal quantity Q_{nom} in g or ml	Value of T in % of Q_{nom}	Value of T in g or ml
0 to 50	9	-
50 to 100	-	4.5
100 to 200	4.5	-
200 to 300	-	9
300 to 500	3	-
500 to 1'000	-	15
1'000 to 10'000	1.5	-
10'000 to 15'000	-	150
Above 15'000	1	-

- (1) **Average requirements:** On average, the quantity in prepackages shall at least be equal to the nominal quantity Q_{nom}
- (2) **Individual requirements:** Only a small percentage of the prepackages (typ. 2.5 %) are allowed to have a quantity between $(Q_{nom} - 2T)$ and $(Q_{nom} - T)$, called *T1* error.
- (3) No prepackage shall have a quantity less than $Q_{nom} - 2T$ (referred to as *T2* error).

OIML R 87: Metrological requirements (2)

Nominal quantity of product in length	Percent of Q_{Nom}
$Q_{nom} \leq 5 m$	No tolerable deficiency allowed
$Q_{nom} > 5 m$	2

Nominal quantity of product in area	Percent of Q_{Nom}
All Q_{nom}	3

Nominal quantity of product in count	Percent of Q_{Nom}
$Q_{nom} \leq 50 items$	No tolerable deficiency allowed
$Q_{nom} > 50 items$	1 (rounded up)

OIML R 87: Metrological requirements (3)

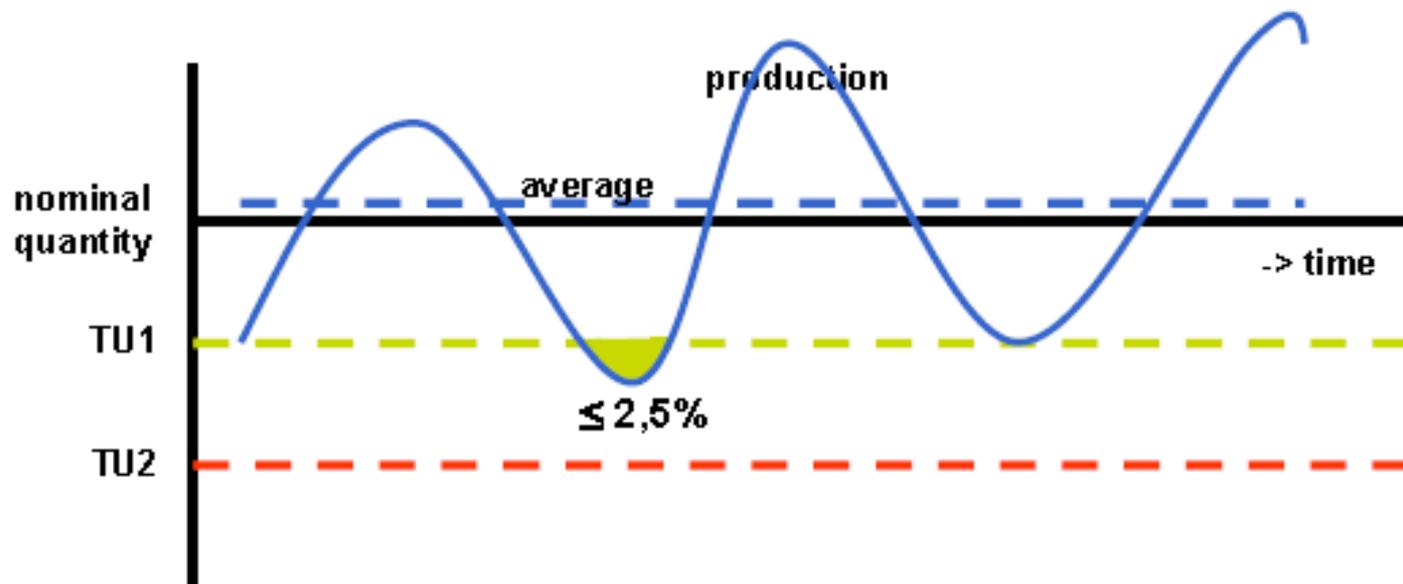
The so called «3-packers rules»:

- In average, the quantity of product in prepackages shall not be less than the declared nominal quantity Q_{nom}
- Only a small amount of prepackages (typ. 2.5 %) are allowed to show deficiencies between $T1$ and $T2$.
- No prepackage shall have a quantity less than $Q_{nom} - 2T$ (referred to as $T2$ error)



OIML R 87: Metrological requirements (4)

- The average quantity in a batch (1 h production) shall not be less than Q_{nom} .
- TU1 (or T1) : only few prepackages (typ. $\leq 2.5\%$) shall exceed a T1 error
- TU2 (or T2): no prepackages allowed to exceed the TU2 (or T2) error.



OIML R 87: Statistical approach

The tests are carried out by statistical means, based on random sampling of an hourly lot of production with lot size N and sample size n .

Average requirements for sampled lots:

- The probability of incorrectly rejecting an inspection lot satisfying equation $\mu \geq Q_{nom}$ shall be no more than 0.5 % (Producer's risk, PR).
- The probability of correctly rejection an inspection lot shall be at least 90 % (Consumer's risk, CR)

Individual requirements for sampled lots:

- The probability of incorrectly rejecting a lot with 2.5 % of the prepackages having $T1$ or $T2$ errors shall be smaller than 5 % (PR).
- The probability of correctly rejecting an inspection lot with 9 % of the prepackages having $T1$ or $T2$ errors shall be at least 90 % (CR).

OIML R 87: Test of average requirements (1)

Reject the lot if $e_{ave}/s + SCF < 0$ with $e_{ave} = \frac{1}{n} \sum_{i=1}^n e_i$

e_{ave} is the average of errors and e_i are the individual prepackage errors defined by $e_i = q_i - Q_{nom}$, taking into account their signs, n is the number of prepackages of the sample size, s is the sample standard deviation of the individual errors, and SCF is the Sample Correction Factor.

Please note the difference between e_{ave} and E_{ave}

e_{ave} : means average error for prepackages of sample size n

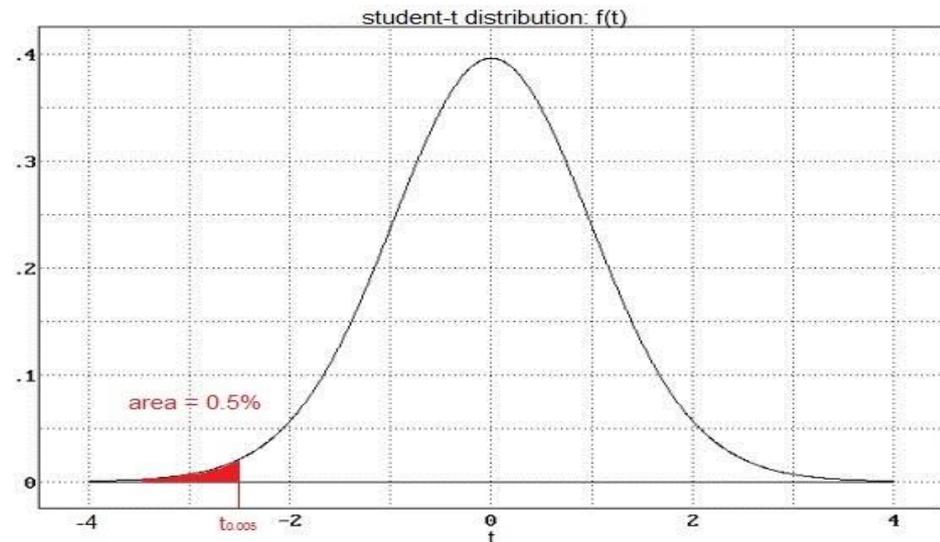
E_{ave} : means average error for prepackages of lot size N

OIML R 87: Test of average requirements (2)

The SCF for the defined lot size N and sample size n is calculated using the formula:

$$SCF = - \frac{t_{0.005, n-1}}{\sqrt{\frac{n(N-1)}{N-n}}}$$

$t_{0.005, n-1}$ is the quantile of the Student's t inverse cumulative function with probability $p = 0.005$ and degree of freedom $f = n-1$



$(N-n)/(N-1)$ is the so called finite population correction factor

OIML R 87: Test of average requirements (3)

Please note:

$$e_{ave}/s + SCF < 0 \quad \text{with} \quad e_{ave} = \frac{1}{n} \sum_{i=1}^n e_i$$

can also be formulated or rewritten in the following way (which is often easier to be understand):

Reject the lot if: $q_{ave} < Q_{Nom} - SCF \cdot s$ with

q_{ave} : mean value of actual quantities q_i in a sample n

Q_{Nom} : Nominal quantity declared on the label

OIML R 87: calculations of SCF (1)

Example of calculation of the sample correction factor SCF by using a lot size of $N = 100$ and sample size $n = 49$

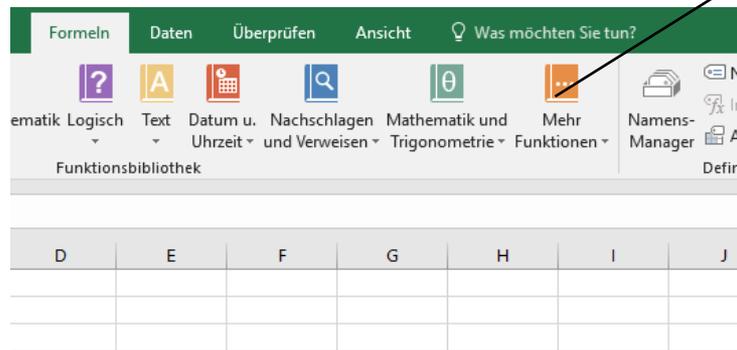
Procedure:

1. Use EXCEL and calculate $t_{p,n-1}$ with $p = 0.005$ and $n = 49$

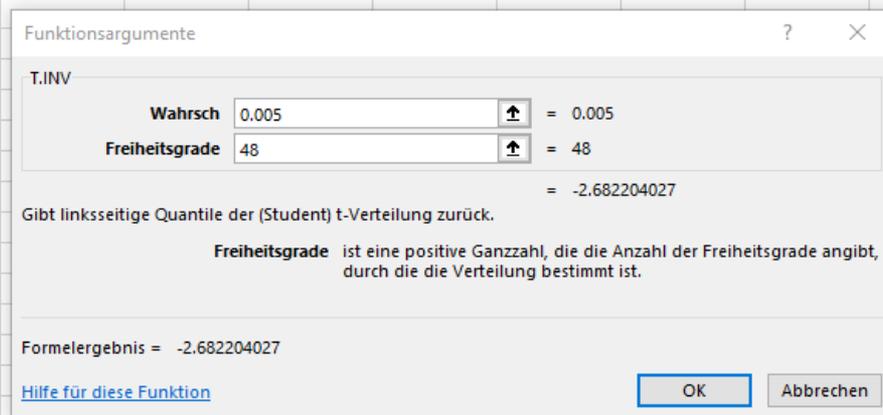
2. Calculate SCF according:
$$SCF = - \frac{t_{0.005,n-1}}{\sqrt{\frac{n(N-1)}{N-n}}}$$

OIML R 87: calculations of SCF (2)

Calculate $t_{p,n-1}$ with $p = 0.005$ and $n = 49$



choose Statistics
then choose T.INV



Excel calculates the
value of $t_{p,n-1} = - 2.6822$
with $p = 0.5\%$ and $n-1 = 48$

OIML R 87: calculations of SCF (3)

Once the value of $t_{p,n-1}$ has been calculated for the probability $p = 0.5 \%$ and a given sample size n , the sampling correction factor SCF can be calculated using the formula:

$$SCF = - \frac{t_{0.005,n-1}}{\sqrt{\frac{n(N-1)}{N-n}}}.$$

for a given lot size N and sample size n .

In our case, with $N = 100$ and $n = 49$ the value of SCF will be:

$$\mathbf{SCF = 0.275}$$

OIML R 87: calculations of SCF (4)

- **Example 1**
Calculate SCF for $N = 500$ and $n = 81$
- **Example 2**
Calculate SCF for $N = 20'000$ and $n = 98$
- **Example 3**
Calculate SCF for $N = 50'000$ and $n = 98$

OIML R 87: Test of individual requirements

The test for the two individual requirements is realized by performing statistics for the values of n_{T1} and n_{T2} .

- The number n_{T1} is the maximum number of samples in the sample size n being allowed for accepting the lot under the assumption that only 2.5 % of prepackages of the lot with size N are allowed to have a T1 error.
- The lot has to be rejected for n_{T2} being larger than zero, this means no prepackage shall have a quantity less than $Q_{nom} - 2T$.

OIML R 87: Single step sampling plan

Sampling plan for lot sizes N based on the statistical requirements (average and individual requirements):

Inspection lot size N	Sample size n	SCF	Number of allowed $T1$ errors n_{T1}
$N \leq 20$	100 % inspection	NA	0
40	32	0.22	1
60	35	0.30	1
80	47	0.25	2
100	49	0.28	2
200	64	0.27	3
300	67	0.29	3
400	81	0.26	4
500	81	0.27	4
600 – 30'000	98	0.24 – 0.26	5
> 30'000	98	0.27	5

OIML R 87: Multistep sampling plan

The stepwise sampling plan is a new feature in the OIML R 87 Edition 2016

Why a stepwise sampling plan?

- Decreases the burden on packers by requiring fewer samples to be taken for control inspections.
- Saves time and costs for inspectors as well as packers since fewer prepackages have to be opened and destroyed for control purposes
- Particularly beneficial for inspectors in checking high volume production of prepacked goods

OIML R 87: Multistep sampling plan for lot size $N = 120$ and 4 steps

Step number	Cumulative sample size n	SCF	Number of allowed $T1$ errors n_{T1}
1	35	0.39	0
2	50	0.29	1
3	60	0.24	2
4	75	0.19	3

OIML R 87: Multistep sampling plan for lot size $N = 120$ and 4 steps

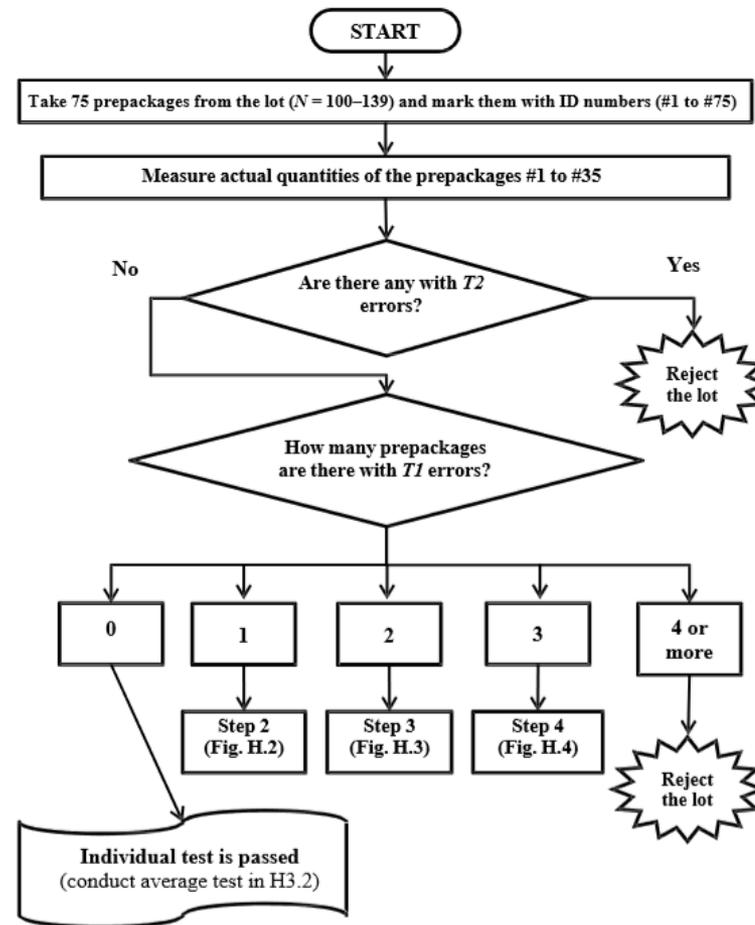


Figure H.1 Stepwise sampling method to test the individual requirement for a lot size $N = 100-139$
Step 1: Start the entire procedure

OIML R 87: Multistep sampling plan for lot size $N = 50'000$ and 7 steps

Step number	Cumulative sample size n	SCF	Number of allowed $T1$ errors n_{T1}
1	40	0.43	0
2	55	0.36	1
3	70	0.32	2
4	95	0.27	3
5	105	0.26	4
6	120	0.24	5
7	135	0.22	6

OIML R 87: Multistep sampling plan

Typical Application:

- For producers with high hourly production rate of prepackages (eg Chocolate manufacturer)
- Manufacturer with low risk of underfilling prepackages
- Checks of prepackages at stock or warehouses by legal metrology officials





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Thank you very much for your attention